

Improving Group Transit Schemes to Minimize Negative Effects of Maritime Piracy

Ondřej Vaněk, Ondřej Hrstka, and Michal Pěchouček

Abstract—Contemporary maritime piracy around the Horn of Africa presents a serious threat to the global shipping industry. A number of countermeasures was deployed to minimize the probability of a successful ship hijack, one of them being the establishment of the International Recommended Transit Corridor (IRTC). Currently, all ships transiting the Gulf of Aden are recommended to follow the IRTC and take part in group transit schemes—prescribed fixed schedules stating a time of arrival to the beginning of the corridor and a speed at which to sail through the corridor.

We provide number of contributions that improve the group transit schemes: we formalize the grouping problem, we design an efficient algorithm able to compute optimal fixed group transit schemes with respect to the distribution of ships' speeds, we provide a real-world dataset with speeds of ships transiting the IRTC, we compare the optimal fixed schedules to the currently deployed schedule and quantify possible savings. Additionally, we propose on-demand group transit schemes—customized schedules for a group of arriving ships—that take into account speeds, risk aversion and actual positions of arriving ships. We formulate the problem of the optimal on-demand grouping as a bi-objective mixed integer program, and we compute a set of Pareto optimal solutions. We evaluate the scalability of the approach, the structure of the solution and quantify an improvement over the current group transit scheme with respect to the number of ships grouped as well as the time saved.

Index Terms—transportation, marine navigation, security, optimization, mathematical programming

I. INTRODUCTION

Contemporary maritime piracy presents a serious threat to the global shipping industry with its annual total costs estimated at up to US\$7bn [1]. The recent spike of piracy in the Indian Ocean and in the Gulf of Aden required an international response in a form of international naval forces deployed in the area. They provide protection to the transiting merchant ships and their presence significantly reduces the success rate of pirate attacks. To provide the best protection of merchant ships possible, International Recommended Transit Corridor (IRTC) was established in the Gulf of Aden which aligns ships along a single lane and a *group transit scheme* (GTS) [2] was proposed to group ships inside the corridor into groups with predefined speeds for easier escort with a limited number of patrolling naval ships.

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Manuscript received ...; revised ...

This work was supported by the U.S. Office of Naval Research (grant no. N000140910537) and by the Czech Ministry of Education, Youth and Sports (grant no. LH11051).

Current GTS is described by a schedule which is defined by 5 *speed levels*—creating groups transiting the corridor at speeds of 10, 12, 14, 16 and 18 knots—and by specific hours at which the groups leave from each entry point of the corridor. To participate in the group transit, a ship slows down to the closest predefined speed level and arrives at one entry point of the corridor at the predefined hour. Groups are disbanded after the corridor transit. Ships sailing faster than 20 knots are recommended to sail at their normal speed without slowing down and participating in the GTS.

The grouping mechanism is—while reducing probability of a successful attack—causing a significant delay: ships participating in the group transit have to slow down prior the corridor transit to arrive to the entry point of the corridor at the predefined hour and they have to sail at the speed of the group throughout the corridor. This delay can be directly translated into increased costs of shipping.

In the first set of contributions, we focus on the direct improvement of the currently deployed GTS with respect to delay minimization while keeping the predefined number of speed levels: (i) we provide a formalization capturing properties of the problem; (ii) we design a scalable algorithm able to compute optimal schedules for any number of speed levels; (iii) we compute optimal schedules for a various number of speed levels and compare them with the currently deployed GTS. Finally, (iv) we create a dataset for evaluation of various GTS schedules by collecting real-world records of ships' trajectories; the dataset is freely-available and contains average speeds of ships transiting the Gulf of Aden.

The second set of contributions is focused on the proposition of a more advanced concept deployable in the Gulf of Aden—*on-demand group transit schemes*. Given its simplicity, current GTS has a major disadvantage: its fixed nature does not take into account speeds of ships approaching each day and it potentially causes longer delays than a schedule designed specifically only for the approaching ships. We design an on-demand grouping mechanism for a set of approaching ships which is based on their actual needs and restrictions rather than on a fixed schedule: (i) we formalize the problem of the on-demand grouping of approaching ships; (ii) we propose a set of mathematical programs able to compute optimal on-demand groupings for different constraint sets and (iii) we evaluate properties of proposed mathematical programs and quality of computed solutions.

Having the algorithms for the optimum fixed and for the on-demand GTS, we compare various solutions with the currently deployed GTS. We show that the currently deployed solution is not far from the optimum fixed schedule for the same number

of speed levels, however, significant savings can be achieved by extending the fixed GTS to 6 speed levels. After exploring the structure and scalability of the on-demand GTS, we also compare it with the currently deployed fixed schedule and show the superiority of the on-demand solutions in terms of the time saved and the number of ships successfully grouped.

II. RELATED WORK

Maritime piracy in the Indian Ocean is a complex phenomenon and affects the maritime transportation system on many levels. If we narrow the focus on the intelligent transportation systems, we find a number of simulation-based tools available to assess effectiveness of countermeasures currently deployed in the Gulf of Aden to lessen the negative impact of piracy on the maritime transport.

Bruzzone et. al[3] model piracy around the Gulf of Aden using the discrete-event simulator PANOPEA and focus on evaluating the efficiency and effectiveness of different Command and Control models. Tilis [4] employs the MANA agent-based modeling framework [5] to identify key factors affecting the escort of vulnerable merchant vessels through the Gulf of Aden. The escorting scenario is modeled on a tactical level, focusing on positioning of individual ships and protection of one group of merchant vessels. Finally, AgentC framework [6] models the maritime piracy in the complete Indian Ocean, allowing evaluation of the impact of various grouping mechanisms on the dynamics of the merchant traffic in a wider area [7]. However, any of these tools do not allow directly to *design* the countermeasures, their primary focus is on validation. We aim to fill this gap and propose a tool for the design of various grouping mechanisms.

Currently deployed GTS in the Gulf of Aden, together with the description of the International Recommended Transit Corridor, is well explained on the operational level by Intertanko [2]; however, the computation of the currently used times and speeds for groups is not described—to our best knowledge—in any of the public sources.

The optimization of the fixed GTS in the Gulf of Aden has been recently approached by Hrstka et. al [8], where the authors derived a formal model of the problem and proposed a set of algorithms able to compute optimal schedules, however, the approach was not scalable even though the problem is solvable in polynomial time. In the first part of this paper, we extend this work by proposing more compact formal model and we design a new scalable algorithm able to compute optimal fixed schedules for tens of groups in seconds.

On-demand GTS is a new paradigm for the IRTC group transit, and—due to its specific set of constraints—no model is directly applicable to the problem. Different techniques have been used to solve related problems, however, none of the approaches is directly reusable and we need to propose a novel formal model of the problem. Here, we describe the most relevant solutions used in other domains to group agents with various requests.

From the domain and approach perspective, most relevant problems are Convoy Formation, Routing, and Movement Problems [9] which involve routing and scheduling military

or emergency rescue convoys with strategic constraints. Montana et al. [10] solve a typical convoy moving problem: the minimization of total movement time of convoys moving in a directed graph, subject to a set of following constraints: the convoys do not stop en-route (same constraint), they do not cross each other, they have the same speed, and others, less relevant constraints to our problem. We allow groups to have different speeds and to cross each other during the transit. Montana et al. additionally focus on convoy scheduling, i.e. the grouping of trucks into one convoy, posing constraints on the size of the group (similar to our formulation) and what type of load the trucks transport (which is irrelevant in our problem).

Typical variants of the convoy movement problem are considered to be NP-hard [11] and operation research techniques, such as mathematical modeling, are typically used to formalize the problem. Montana et al. [10] use genetic algorithms to find optimum convoy schedules. Chardaire et al. [12] model the problem as an integer program; they solve large-scale instances by using Lagrangian relaxation and evaluation of the dual function and obtain heuristic solutions for the original formulation. In our work, we use integer programming to capture the structure of the problem as well, however, we are interested only in optimal solutions. Kumar et al. [13] address a bi-criteria version of the convoy movement problem with minimizing the total travel time and travel span as objectives. We capture our objectives as a bi-criterion function as well, however, we look at travel time and risk taken and use a different solution approach.

Previous work is motivated mainly by military and emergency rescue domains. We can find similar work in classical transportation domain as well, where similar problem arises on public highways. Khan and Boloni [14] formalize the problem of vehicles joining and leaving a convoy while having an upper and a lower speed limit and an acceptable utility for being in a convoy. The formulation is similar to our problem; however, the work does not consider any temporal constraints. Algorithms are based on coalition formation with non-transferable utility techniques and solutions reflect the optimum decision from the vehicle’s point of view, not from the social welfare maximization perspective.

One of the frequently solved problems in urban transportation is the car pooling [15] problem consisting of defining subsets of passengers that will share cars and the paths the cars should follow, so that number of passengers per car is maximized and the sum of the path costs is minimized. The goal is to plan a set of minimum cost vehicle routes capable of serving as many passengers as possible, under a set of constraints arising from the spatial distribution of the problem. The special case of the car pooling problem with all cars being identical is called a Dial-a-Ride Problem [16]. Both problems can be solved heuristically or exactly using integer programming techniques. Again, even though our problem can be technically reduced to planning on a single-edge graph, the methods here cannot be directly reused, as we pose different constraints on the groups and we cannot group arbitrary agents into a single group.

In each of the problems presented, we can find a subset of

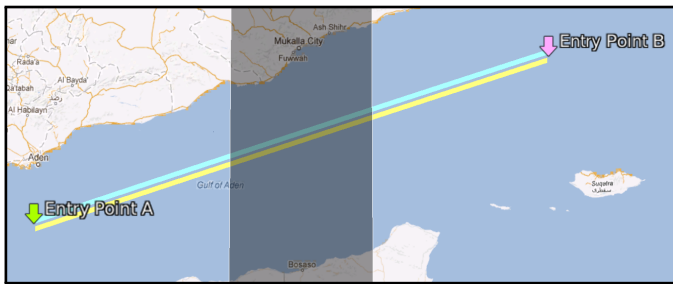


Fig. 1. IRTC Corridor in the Gulf of Aden. Ships enter the gulf area from both sides. The most dangerous area in the middle (denoted by the dark zone) is best to be crossed at night.

constraints which are valid and a subset of constraints that do not hold for the on-demand GTS formation. In Section V we present a formal model of the problem and similarly to work presented above, we use mathematical programming to find an optimal solution.

III. PROBLEM DESCRIPTION

Gulf of Aden is a narrow area north from coast of Somalia with dense merchant traffic in both directions, mostly transporting goods and oil from Asia to Europe. Current spike in Somali-based piracy poses a serious threat to merchant ships transiting the area of the Indian Ocean and the Gulf of Aden – hundreds of hijack attempts were reported from 2008 till today and tens of ships were hijacked every year.

A. International Recommended Transit Corridor

International naval forces were deployed in the area to protect the merchant ships and International Recommended Transit Corridor (IRTC) was established to align the traffic into a two lanes – separating East bound and West bound traffic – for easier protection (see Figure 1). The East bound lane begins at the entry point A and is oriented along a straight line course of 72 degrees. The West bound lane begins at the entry point B and is oriented along a straight line course of 252 degrees [2]. The most dangerous area for the transit is approximately in the middle of the corridor and is recommended to be crossed at night (the dark area in Figure 1).

B. Group Transit Scheme

In August 2010, a group transit scheme was introduced to further reduce the risk of pirate attacks (illustrated in detail in [2]). Gulf of Aden Group Transits are designed to group ships into different speed groups in order to exploit additional protection and assurance of traveling in a group. Each group is defined by a *speed level* (i.e., published speed at which all ships belonging to the group sail) and *entry time* at which all the ships belonging to the group has to be at the corridor entry point. There is one transit per day for each speed group (shown in the Table I).

The entry times for different speed groups to enter the IRTC are calculated so that the groups pass through the most dangerous area at night and they ensure that all ships, regardless of speed, are together at dawn. The group transit

TABLE I
GULF OF ADEN GROUP TRANSIT SCHEDULE.

Speed Level	Entry Time for EP A	Entry Time for EP B
10 kn	04:00	18:00
12 kn	08:30	00:01
14 kn	11:30	04:00
16 kn	14:00	08:30
18 kn	16:00	10:00

scheme thus groups the ships on two tiers: first, the ships are grouped according to their speed and second, the groups are grouped again at the most dangerous area in the Gulf of Aden to transit the area at night and close together. This allows the military forces to best position their assets in the area so as to protect ships against piracy and to provide assistance in case of attack.

It is important to note relative simplicity of the GTS schedule: speed levels and times are rounded and relatively easy to remember. However, given that every ship transiting the area has to register with a maritime center that provides necessary information and availability of automatic systems for ship control, we do not see any practical problem in deploying times rounded to minutes and speeds rounded to tenths of knots.

C. Motivation for Improvement

Even though the existing group transit scheme aggregates the traffic and lowers the hijack probability, it can be further optimized to take into account following facts.

(1) Current group transit scheme does not seem to reflect the distribution of speeds of merchant ships. Most of the ships sail at 14 – 17 knots, however, the selection of speed levels in the GTS is rather uniform, resulting into unnecessary increased delay in the Gulf of Aden transit. The fixed GTS can be optimized to take the speed distribution of ships into account. Additionally, different numbers of speed levels should be explored to understand the dependency between the number of speed levels and the delay caused.

(2) Current group transit scheme is *fixed* in advance, i.e. it does not take into account speeds of actual ships arriving into the Gulf. For these ships, a on-demand group transit scheme can be designed which adheres to the constraints given by the second tier grouping (i.e. all groups should transit the most dangerous area together) and which minimizes the delay caused by the group transit scheme only for the arriving ships, better taking into account their actual speeds; additionally, the on-demand group transit scheme should also lower the probability of a ship being left out of the grouping mechanism.

(3) Current group transit scheme does not take into account the risk aversion of individual ships, i.e., if a ship has invested into counter-piracy measures and she is well protected against any attack, she does not have to take part in the group transit.

IV. OPTIMUM FIXED GROUP TRANSIT SCHEME

In this section we formalize the problem of optimal fixed GTS and describe the design of an optimal and scalable algorithm able to compute fixed GTS for any number of speed levels.

The criterion is to minimize the delay caused by the group transit scheme, given the *histogram* of ships' speeds H and the *number of speed levels* S as the input to the algorithm¹. The histogram is created from a dataset with the bin width d kn. The domain of the histogram H forms an ordered set \mathbf{D} where $v \in \mathbf{D}$ is the speed of ships in the respective bin and $H(v)$ is the number of ships with the speed v . Our solution is based on finding a combination of S speed levels with minimal sum of ships' delays. This can be also viewed as partitioning histogram H into S parts (see Figure 5 for exemplar histograms). Without loss of generality, we describe a solution for a single lane in the corridor.

A. Formal Model

We search for a set of S groups $\mathbf{G} = \{g_1, \dots, g_S\}$, $S < |\mathbf{D}|$. Group g_i is defined by a discrete interval $[g_i, \bar{g}_i] \subset \mathbf{D}$, where \underline{g}_i is the transit speed of the group g_i and \bar{g}_i is the maximum allowed speed of a ship in g_i . This means, that each ship in group g_i with a speed $v \in [g_i, \bar{g}_i]$ must reduce her speed to \underline{g}_i . Group intervals are disjunct and their union is equal to \mathbf{D} .

The delay for one ship at speed v is defined as $L(1/\underline{g}_i - 1/v)$, where \underline{g}_i is the speed of group, to which the ship belongs and L is the length of transit corridor. The delay for all ships in one group g_i is then defined as sum of ships' delays:

$$F(g_i) = \sum_{v \in [g_i, \bar{g}_i]} H(v) \cdot L \left(\frac{1}{\underline{g}_i} - \frac{1}{v} \right) \quad (1)$$

As defined, the criterion for optimization task is the minimization of ships' delays. We thus aim to minimize the sum of all groups' delays:

$$\min \sum_{i=1}^S F(g_i) \quad (2)$$

$$\text{s.t.} \quad \bigcap \mathbf{G} = \emptyset \quad (3)$$

$$\bigcup \mathbf{G} = \mathbf{D} \quad (4)$$

B. Solution Approach

Optimum fixed group transit scheme problem can be translated into a problem of the minimum cost path search in a directed acyclic graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each node $n(\underline{g}_i, \bar{g}_i, i) \in \mathcal{V}$ represents i -th speed group g_i . Due to the speed restriction imposed on \underline{g}_i and \bar{g}_i (i.e., $\underline{g}_i \leq \bar{g}_i$), the set of nodes \mathcal{V} is then set of all nodes:

$$\mathcal{V} = \{n(\underline{g}_i, \bar{g}_i, i) | \underline{g}_i \leq \bar{g}_i, i = 1, \dots, S\} \quad (5)$$

We define cost of a node $n(\underline{g}_i, \bar{g}_i, i)$ as the value of the function $F(g_i)$.

The edge set \mathcal{E} is defined as follows:

$$\mathcal{E} = \{e(n(\underline{g}_i, \bar{g}_i, i), n(\underline{g}_j, \bar{g}_j, j)) | j = i + 1, \underline{g}_j = \bar{g}_i + d\} \quad (6)$$

¹The number of speed levels is given by the authorities supervising the transit of the area, such as Maritime Security Centre Horn of Africa (<http://www.mschoa.org>), and it is based on the number of naval vessels operating in the area.

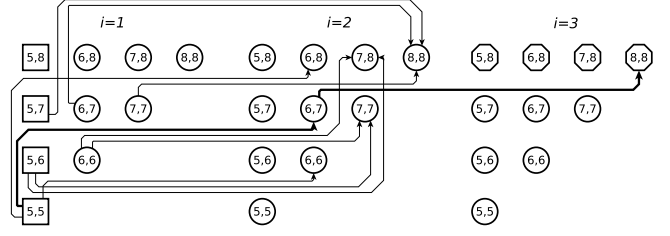


Fig. 2. Example of graph \mathcal{G} for $\mathbf{D} = \{5, 6, 7, 8\}$. Nodes labels are in format $(\underline{g}_i, \bar{g}_i)$, start nodes are rectangular, terminal nodes are octagonal. For better readability, only a subset of periodically repeating edges is depicted. Edges with thick line represent the shortest path (i.e. the optimal solution) in graph.

i.e., the edge set \mathcal{E} contains only edges that connect nodes with coordinate i to nodes with coordinate $i + 1$ and additionally, groups represented by nodes $n(\underline{g}_i, \bar{g}_i, i)$ and $n(\underline{g}_j, \bar{g}_j, j)$ connected by an edge meet following properties: $[\underline{g}_i, \bar{g}_i] \cup [\underline{g}_j, \bar{g}_j] = [\underline{g}_i, \bar{g}_j]$ and $[\underline{g}_i, \bar{g}_i] \cap [\underline{g}_j, \bar{g}_j] = \emptyset$.

The solution is the minimal cost path from origin node set $\mathcal{O} = \{n(\underline{g}_i, \bar{g}_i, i) | \underline{g}_i = \min(\mathbf{D}), i = 1\}$, to the terminal node set $\mathcal{T} = \{n(\underline{g}_i, \bar{g}_i, i) | \bar{g}_i = \max(\mathbf{D}), i = S\}$.

Note that every path from \mathcal{O} to \mathcal{T} contains – due to the nature of the graph construction – exactly one node per each layer which guarantees fixed length of the path.

The special construction of the graph now allows us to use any algorithm for finding the minimum cost path between the start node set \mathcal{O} and terminal node set \mathcal{T} in form of an ordered set of nodes

$$\mathcal{V}^* = \{n_1(\underline{g}_1, \bar{g}_1, 1), \dots, n_S(\underline{g}_S, \bar{g}_S, S)\}$$

An algorithm for finding a minimal cost path in a directed acyclic graph (which is our case) – described in [17] – has a linear complexity $O(|\mathcal{V}|)$.

Example: We have ships' speed histogram as follows: The ships speeds set $\mathbf{D} = \{5, 6, 7, 8\}$, respective counts of ships of given speed are $H(v)_{v \in \mathbf{D}} = \{5, 10, 4, 6\}$, and the number of groups is $S = 3$. The graph \mathcal{G} for this problem is depicted in Figure 2. For better readability, only edges between nodes with $i = 1$ and $i = 2$ are depicted. Nodes are labeled as $(\underline{g}_i, \bar{g}_i)$, omitting the group index for better readability. The start nodes are rectangular and the terminal nodes are octagonal. The optimal solution for this example is drawn with thick edges and consists of the following groups:

$$\mathbf{G}^* = \{g_1, g_2, g_3\} = \{[5, 5], [6, 7], [8, 8]\}$$

causing delay of 5 minutes and 42 seconds per kilometer.

To show that the approach described above finds an optimal fixed group transit scheme, we have to show, that the minimum cost path found represents the fixed group transit scheme.

Theorem 1. The solution found by the described algorithm is the solution of the fixed schedule optimization problem defined by (2) – (4).

Proof: The sum of costs of nodes \mathcal{V}^* representing the

shortest path is equal to the problem criterion (2):

$$\sum_{n(\underline{g}_i, \overline{g}_i, i)}^{\mathcal{V}^*} F(g_i) = \sum_{i=1}^S F(g_i) \quad (7)$$

because the path found in graph has minimal sum of the nodes' costs it also satisfies the minimization criterion (2).

Constraint (3) is not violated due to the graph topology. For each edge $e(n_i(\underline{g}_i, \overline{g}_i, i), n_j(\underline{g}_j, \overline{g}_j, j))$ holds that $\overline{g}_i < \underline{g}_j$. Hence there are no overlapping speed groups in the solution.

Constraint (4) is not violated because for any two speed intervals $[\underline{g}_i, \overline{g}_i]$ and $[\underline{g}_j, \overline{g}_j]$ represented by two subsequent nodes n_i and n_j from the solution \mathcal{V}^* holds that their union is $[\underline{g}_i, \overline{g}_j]$ and their intersection is empty. For the first speed group represented by node n_1 , the interval lower bound is $\underline{g}_1 = \min(\mathbf{D})$ and for the last speed group represented by node n_s , the interval upper bound is $\overline{g}_s = \max(\mathbf{D})$. Hence the union of the intervals represented by nodes in the solution \mathcal{V}^* is \mathbf{D} . ■

Additionally, it is necessary to show that there always exists a solution for S groups that is better than any solution for less groups, so we do not have to consider paths shorter than those leading from \mathcal{O} to \mathcal{T} . First, let us prove a simpler theorem and use induction to show this property.

Theorem 2. There always exists a solution for S groups that is better than any solution for $S - 1$ groups.

Proof: Consider the optimal solution for $S - 1$ groups. If we take arbitrary group $g_i = [\underline{g}_i, \overline{g}_i]$ with more than one speed bin $|\underline{g}_i| > 1$, we can reduce this group by its lowest speed bin \underline{g}_i and create the new speed group $g_i^+ = [\underline{g}_i, \overline{g}_i]$ consisting of this speed bin. The reduced speed group $g_i^- = g_i \setminus \underline{g}_i$ has smaller cost than g_i and the new speed group g_i^+ has cost $F(g_i^+) = 0$. Therefore we have constructed a solution for S groups that is better than the solution for $S - 1$ groups. ■

By induction, a straightforward corollary of Theorem 2 is that the solution for S groups is better than every solution for $\{n | n < S\}$ groups (note that $S < |\mathbf{D}|$).

C. Entry time calculation

The final step of the process is the calculation of the entry time t_i for each group g_i . The entry time is calculated as follows:

$$t_i = U - \frac{|A - \Pi|}{\underline{g}_i} \quad (8)$$

where A denotes the location of the corridor entry point and Π is an aggregation point inside the dangerous area where the groups should meet at the given time U , i.e., at dawn. $|A - \Pi|$ denotes the length of the orthodromic shortest path between the two locations inside the navigable waters.

V. OPTIMUM ON-DEMAND GROUP TRANSIT SCHEME

The on-demand group transit scheme optimization poses a more complex problem than the fixed GTS optimization, as it tackles the grouping problem on the scale of individual ships and thus takes into account constraints imposed by the spatial distribution and by different capabilities of the approaching

ships: for each ship arriving to the entry point of the corridor, we seek an assignment of the ship to a group formed with other arriving vessels; the groups are created on demand and the speed level of each group is dynamically determined by the the slowest vessel in the group, i.e., the speed of a group is equal to the speed of the slowest vessel in the group.

In this problem we deal with a continuous and possibly infinite stream of approaching ships transiting the corridor. This issue can be solved in two ways: (1) design an algorithm which continuously creates groups and adds ships on the fly (using, e.g., the rolling horizon approach or on-line clustering [18]) or (2) divide the stream of approaching merchant ships into clusters and determine assignment of the merchant vessels into groups separately for each cluster.

We will focus on the latter approach, motivated by the specifics of the Gulf of Aden transit: typically, vast majority of the ships arrive from or continue its journey to the Strait of Suez which employs its own traffic system, structuring all ships transiting the strait into groups which transit the strait once per day². This means, that the daily transit of ships through the Strait of Suez forms a cluster for which we design the on-demand group transit scheme.

Another algorithm utilizing on-line clustering of stream of arriving merchant vessels is currently being solved in another research branch and it is not described in this paper.

A. Abstraction

The informal description of the grouping is described in Section III-B and we want the on-demand GTS to follow the same rules of transit. Figure 3 depicts the abstraction of the situation: the ships transit the area from left to right, reaching the *approach zone* first. Only the ships in the approach zone (of length L_a) are considered for grouping.

For a cluster of ships in the approach zone, we define a maximum number of groups S than can be assembled. The groups are established at the entry point A and they follow the corridor (of length L). Additionally, inside the corridor, there is an *aggregation point* Π , in which all the groups have to meet. Moreover, the aggregation point has a specific time of the day U assigned, at which the groups have to arrive at the point³. The last two conditions impose additional constraints on the problem and can be relaxed in similar models without such requirements.

For a cluster of ships in the approach zone, the computation of the group transit formation is performed at a single time instant τ^4 when the ships are at positions P inside the approach zone and their unconstrained speed is V . We impose minimum approach speed constraint on the arriving vessels—i.e., the vessels cannot slow down under ν knots. Having the specific time of the day U to reach the aggregation point, the ships' positions and the minimum approach speed defined, we can compute all possible date-times at which the

²For details, see Strait of Suez Traffic System website: <http://www.suezcanal.gov.eg/sc.aspx?show=13>.

³In the current fixed GTS, the time is set to 6 am local time.

⁴The computation can be performed, e.g., once per day at 12 am, however, the frequency depends on the length of the approach zone.

TABLE II

PARAMETERS AND VARIABLES OF THE MATHEMATICAL PROGRAM OF THE ON-DEMAND GTS.

Constants	Definition
$S \in \mathbb{N}$	maximum number of groups
$N \in \mathbb{N}$	number of ships in the approach zone
$L \in \mathbb{R}^+$	length of the corridor [nm]
Π	coordinates of the aggregation point [GPS]
A	coordinates of the entry point [GPS]
P_j^τ	position of the j -th ship at time of computation τ [GPS]
$V_j \in \mathbb{R}^+$	speed of the j -th ship (outside of a group) [kn]
$\mathbf{U} \in \mathbb{R}^m$	vector of admissible times to be at the aggregation point Π w.r.t. the minimum approach speed ν and current time τ [h]
$R_j \in [0, 1]$	risk value, expressing the risk aversion of the j -th ship
$\nu \in \mathbb{R}^+$	minimum approach speed of ships [kn]
$\mu \in \mathbb{N}$	minimum group size
$\tau \in \mathbb{R}^+$	time of computation
$\Delta V \in \mathbb{R}^+$	maximum allowed speed difference in one group [kn]

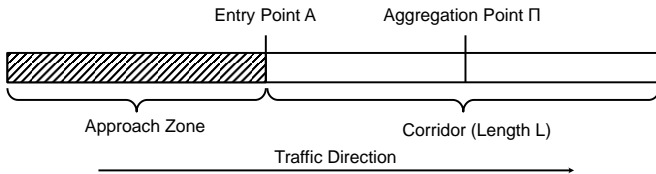


Fig. 3. Abstraction of the on-demand group transit setting.

ships can arrive at the aggregation point, introducing a list of admissible aggregation point arrival date-times \mathbf{U} . In the following sections, \mathbf{U}_k then denotes the k -th date-time in \mathbf{U} and $m = |\mathbf{U}|$.

All predefined or pre-computed constants used throughout the section in the mathematical program are summarized in the Table II and are capital or Greek letters.

B. Mathematical Model

To express the advantage of a ship being in a group, we use a term *risk*. If a ship does not take part in the group transit, she is facing an increased risk of hijack because she does transit the area with a group and she may not obey the required time of transit of the most dangerous area. We model the increased risk with a *risk aversion* $R_j \in [0, 1]$ parameter, set individually for each ship. For $R_j = 0$, the ship does not gain anything by being in a group, for $R_j = 1$, the ship suffers maximum penalty for being left out of the GTS.

We formulate the problem of on-demand group transit as a bi-objective optimization problem with two objective functions: *delay* and *risk*, which are subject to a set of constraints on the schedule properties and ship abilities:

$$\min \quad (\text{delay}(c), \text{risk}(c)) \quad (9)$$

$$\text{s.t.} \quad c \in \mathcal{C} \quad (10)$$

where \mathcal{C} is a set of feasible solutions defined by a set of constraints. We scalarize the multi-objective criterion into a single linear combination of the two functions, weighted by γ parameter:

$$\min \quad D + \gamma \cdot \mathcal{R} \quad (11)$$

where the delay function D is sum of two components $D = D_a + D_t$. D_a is the *approach delay*, caused by the lower speed of the ships in the approach zone and D_t is the *transit delay*, caused by lower speeds of grouped ships during the corridor transit. We consider both delays to have an equal weight, however, a multiplication constant can be added to one delay component to prefer one over the other. \mathcal{R} is the overall risk aggregated over all ships. The solution is not unique due to the counter-going objective functions—delay and risk are inversely proportional. We thus look for a set of Pareto-optimal solutions—i.e. the Pareto front—by varying the γ parameter.

We define the terms in the criterion 11 as:

$$D_a = \sum_{j \in N} \left(\omega_j - \frac{|P_j^\tau - A|}{V_j} \right) \quad (12)$$

$$D_t = \sum_{j \in N} \sum_{i \in S} \sum_{k \in |\mathbf{U}|} \left((\mathbf{U}_k - \omega_j) \cdot \frac{L}{|A - \Pi|} - \frac{L}{V_j} \right) \cdot y_{ijk} \quad (13)$$

$$\mathcal{R} = \sum_{j \in N} R_j \cdot \left(1 - \sum_{i \in S} x_{ij} \right) \quad (14)$$

where $\omega_j \in \mathbb{R}^+$, $x_{ij} \in \{0, 1\}$ and $y_{ijk} \in \{0, 1\}$ are decision variables of the problem: ω_j represents the entry time of the j -th ship at the entry point A ; y_{ijk} is set to 1 if the j -th ship in the i -th group would sail through the aggregation point Π at time \mathbf{U}_k ; x_{ij} is set to 1 if the j -th ship belongs to the i -th group.

Equation (12) defines the overall approach delay summed over all vessels: without the grouping mechanism, the time required for the j -th ship to reach the entry point A is given by the term $\frac{|P_j^\tau - A|}{V_j}$; with the grouping mechanism in place, the time required is ω_j .

Equation (13) sums the transit delay over all ships: each ship has assigned a time \mathbf{U}_k of passing the aggregation point Π through the variable y_{ijk} . Without the grouping, the time required to transit the corridor is given by $\frac{L}{V_j}$. With the grouping mechanism in place, the transit time is given by $(\mathbf{U}_k - \omega_j) \cdot \frac{L}{|A - \Pi|}$.

Equation (14) expresses the risk as a sum of individual risks of all ships which are not assigned to any group (the expression $(1 - \sum_{i \in S} x_{ij})$ is 1 if and only if the j -th ship is not assigned to any group).

Once a solution is found, we can directly compute the arrival speed of the j -th ship before the entry point, the assignment of each ship to a particular group, the speed of each group, and the actual number of groups (which can be lower than S , see below) from the decision variables ω_j , y_{ijk} , and x_{ij} .

Criterion (11) is optimized subject to a number of constraints capturing structure of the grouping mechanism. First, we impose a set of constraints for a correct grouping, expressed as:

$$\sum_{i \in S} x_{ij} \leq 1 \quad \forall j \in N \quad (15)$$

$$\sum_{j \in N} x_{ij} \geq \mu \quad \text{OR} \quad \sum_{j \in N} x_{ij} = 0 \quad \forall i \in S \quad (16)$$

Constraint (15) specifies, that each ship can be at most in one group and constraint (16) restricts the group size to be greater than or equal to a pre-specified minimum group size μ or the group size has to be 0 (thus having no ship assigned).

To capture the requirement of forming the group at the entry point A , we introduce variable z_i capturing the entry time of the i -th group with following constraints:

$$\text{IF } x_{ij} = 1 \quad \text{THEN :} \quad \forall i \in S; \forall j \in N$$

$$z_i = \omega_j \quad \text{AND} \quad (17)$$

$$\frac{|P_j^\tau - A|}{V_j} + \tau \leq z_i \leq \frac{|P_j^\tau - A|}{\nu} + \tau \quad (18)$$

i.e. if the j -th ship belongs to the i -th group, then their entry times have to be equal (Constraint (17)). Constraints (18) express the fact, that a group cannot be established earlier than every ship belonging to that group reaches the entry point A , posing a lower bound restriction on z_i . The upper bound of z_i is given by the constraint that no ship violates the minimum approach speed requirement (given by ν)⁵.

Finally, we incorporate the restriction given by the requirement to aggregate the groups at the aggregation point Π :

$$\text{IF } x_{ij} = 1 \quad \text{THEN :} \quad \forall i \in S; \forall j \in N$$

$$\sum_{k \in |\mathcal{U}|} y_{ijk} = 1 \quad \text{AND} \quad (19)$$

$$\frac{|\Pi - A|}{V_j} \leq \left(\sum_{k \in |\mathcal{U}|} y_{ijk} \cdot \mathbf{U}_k \right) - z_i \leq \frac{|\Pi - A|}{V_j - \Delta V} \quad (20)$$

Constraint (19) states that only one time of passing Π is admissible for any ship. The time needed for the i -th group to reach Π is given by $\left(\sum_{k \in |\mathcal{U}|} y_{ijk} \cdot \mathbf{U}_k \right) - z_i$. Constraints (20) state that this time has to be greater than the time required to reach Π by any ship in the group (lower bound); the upper bound is imposed by the requirement of the maximum difference of speeds in one group to be at most ΔV .

The equations above fully capture the problem, however, we can add additional redundant constraints to speed-up the solution process:

$$x_{ij} \neq x_{il} \quad \text{IFF} \quad |V_j - V_l| > 2 \cdot \Delta V$$

$$\forall i \in S; j, l \in N; j \neq l \quad (21)$$

$$x_{ij} \neq x_{il} \quad \text{IFF} \quad et_j > lt_l \quad \text{OR} \quad et_l > lt_j$$

$$\forall i \in S; j, l \in N; j \neq l \quad (22)$$

i.e. no two ships can be in one group, if the difference of their speed is greater than $2 \cdot \Delta V$. Additionally, no two ships can be in one group if the earliest time et_j of one ship to arrive to A is greater than the latest time lt_l when the other ship has to leave A (given by the minimum approach speed requirement) or vice versa. The times are computed as $et_j = |P_j^\tau - A|/V_j$ and $lt_j = |P_j^\tau - A|/\nu$.

⁵It is trivial to account for different minimum approach speeds for each ship by introducing ν_j variable and directly use it in Constraint (18).

Modern solvers typically support both logical and *if-then* constraints. If a conversion to linear constraints is required, standard techniques from the field of Operations Research can be used (see, e.g., Hooker [19]).

C. On-demand Group Transit Scheme Relaxations

Having the full problem formulation, we can relax or restrict any of the constraints to customize the on-demand GTS. The following variations with minor modifications as well as with more fundamental ones — removing restrictions posed by the the aggregation point Π and/or by the approach buffer (i.e. the groups are assembled directly at the entry point A) — have been considered by the authors:

1) *Mandatory grouping*: We modify Constraint (15) to account for a mandatory assignment of every ship into any group, i.e. $\sum_{i \in S} x_{ij} = 1, \forall j \in N$. Having this restriction, the risk summand in the criterion function is redundant and can be left out. However, the problem cannot be always feasible, due to the requirements on the minimum approach speed (Constraint (18)) and maximum speed difference in one group (Constraint (20)). These constraints have to be either relaxed or the program has to be solved multiple times while decreasing the minimum allowed approach speed ν and/or increasing maximum speed difference ΔV .

2) *Group size limit*: We can additionally limit the size of the group to be at most η ($\eta \geq \mu$) by introducing constraint $\sum_{j \in N} x_{ij} \leq \eta, \forall i \in S$. In this case, the number of groups should be proportionally increased to create enough groups and not to be penalized for the risk of ships that cannot be placed in any group because of this constraint.

3) *No Aggregation Point*: By relaxing Constraints (20) and (19), we do not pose any aggregation requirements on the group transit scheme. As a direct consequence, all groups sail at the group speed equal to the speed of the slowest ship of the group. To reflect these facts in the mathematical model, we consider explicit group speeds which are not linked to the aggregation point. We reformulate the transit delay D_t to:

$$D_t = \sum_{j \in n} \sigma_j \cdot L - \frac{L}{V_j} \quad (23)$$

where σ_j is the *inverse* transit speed of the j -th ship in a group (The variables are defined as inverse to keep the mathematical formulations linear); the actual speed of the j -th ship in a group is thus equal to $\frac{1}{\sigma_j}$. The criterion is subject to the following constraints:

$$\text{if } x_{ij} = 1 \quad \text{then :} \quad \forall i \in S, j \in N$$

$$\sigma_j = g_i \quad (24)$$

$$\frac{1}{V_j} \leq g_i \leq \frac{1}{(\sigma_j - \Delta V)} \quad (25)$$

$$g_i \in \mathbb{R}^+ \quad \sigma_j \in \mathbb{R}^+ \quad (26)$$

Constraint (24) links the inverse speed of the j -th ship σ_j to the inverse speed of the i -th group g_i . Constraints (25) restrict the group speed g_i to be lower than the speed of the slowest ship in the group and allow a group to have ships with maximum speed difference at most ΔV (similar to Constraints (20)).

4) *No Approach Zone*: For some on-demand group transit schemes, the consideration of an approach zone is not required, i.e. we consider all ships to be at point A at the beginning of the grouping, similar to the Convoy Scheduling Problem formulation. We can then relax Constraints (18). However, it is recommended to leave variables z_i (Constraints (20)) to allow groups to start their route with a delay z_i . Otherwise, if the meeting times U_k are too sparse, a reasonable solution may not be found.

5) *No Aggregation Point, No Approach Zone*: Finally, we can relax the restriction on the aggregation inside the corridor and consider the ships to be ready at the entry point A . The delay caused is given only by the transit delay D_t in form of the Equation (23), which is restricted by Constraints (24) – (26). Together with Constraints (15) – (16), they form a complete constraint set for this problem.

VI. GROUP TRANSIT SCHEMES EVALUATION

We evaluate the quality of solutions of both fixed schedule and on-demand GTS optimization problems. We use both synthetic and real-world data and we are interested in the structure of the solution, in the relative improvement against current group transit scheme, and in the scalability of algorithms for on-demand grouping. The algorithms were evaluated on a Quad-core 64-bit PC with 4GB available RAM; the implementation was done in Java 1.7 and we used CPLEX 12.3 to solve the mathematical programs.

A. Datasets

One of the main contributions of this work is the extraction of a real-world dataset containing data for evaluation of the optimality of various group transit schemes. We have also created two synthetic datasets with typical distributions representing maximum entropy (minimum prior information) approach—a uniform distribution and a normal distribution of speeds. All datasets comprise of a set of real numbers between 10.0 and 20.0, representing the speed (measured in knots) of ships moving through the Gulf of Aden that are subject to the group transit scheme. The datasets are available to the reader (at <http://agents.cz/projects/agentc/dataset>):

Real-world—Dataset containing 2366 values of ships' speeds from real-world records of ships transiting the Gulf of Aden. To create the dataset, we have collected a sample of AIS (Automated Identification System) data available through the VesselTracker website⁶ capturing traffic through the Gulf of Aden in 2008 (prior the establishment of the group transit scheme). AIS records are data samples from an automated tracking system used for identifying and locating ships. AIS record contains (among identification details) a sequence of locations of a ship (in a GPS format) annotated with time stamps. From such a sequence, we were able to estimate the average speed of the ship by averaging the speeds between any two subsequent locations inside the Gulf of Aden. Note that speeds of many ships could not be reconstructed because many

⁶VesselTracker website: <http://www.vesseltracker.com/>. Unfortunately, the AIS data were not publicly available in 2013 anymore.

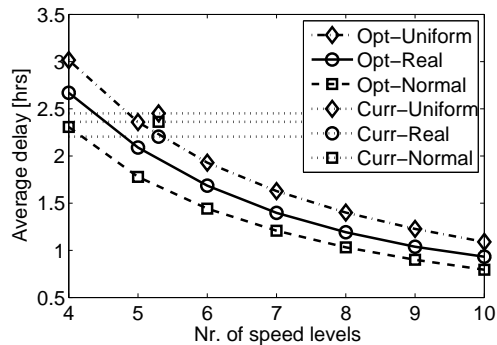


Fig. 4. The delay of transit through the corridor caused by the fixed GTS for various distributions of ship speeds. The values for the current GTS (5 speed levels) are horizontally shifted for better readability.

ships were turning the AIS system during the transit of the Gulf of Aden off. The size of the dataset correspond roughly to 10% of yearly traffic through the Gulf of Aden [20]. The dataset is fully anonymized and does not contain any personal or commercial information.

Uniform—Dataset containing 10000 values sampled from a uniform distribution $U(10.0, 20.0)$.

Normal—Dataset containing 10000 values sampled from a normal distribution $N(15.0, \frac{5}{3})$, i.e. with mean value $\mu = 15.0$ and with standard deviation $\sigma = \frac{5}{3}$. Then, 99.7% of the speeds lie inside the $[10.0, 20.0]$ bounds. Values outside the interval were not added to the dataset.

B. Fixed GTS Evaluation

The fixed GTS optimization directly improves schedules deployed in the real world, the aim of the evaluation is thus a comparison of quality with the current schedule and an exploration of the structure of the solution. We evaluate the algorithm on all three datasets.

1) *Optimality*: The delay of transit through the corridor (without the approach delay) caused by the optimal fixed GTS for all datasets is depicted in Figure 4. We have computed the optimal schedule for each dataset while varying the number of speed levels. The average delay per ship is decreased when increasing the number of speed levels. Observe that for one speed level, we get a different delay for each dataset, i.e. our algorithm is able to exploit the structure of the data.

Additionally, we have computed the average delay for the current GTS on each data set (depicted by dotted lines). The current GTS causes the smallest delay on the real-world dataset; the minimum difference between current GTS-generated delay and optimum GTS-generated delay is observable for the uniform and real-world dataset, giving us insights into the original process of current schedule design.

2) *Structure of the Solution*: Optimal solutions for five, six and eight speed levels for uniform, normal and real-world datasets are depicted in Figure 5. Compared to the uniform spacing of speed levels in the current GTS, all optimal solutions suggest, that the spacing of speed levels should be non-uniform and reflect the greater delay of slower ships (i.e., spacing should be in general tighter for lower speeds—observable in solutions for the uniform speed distribution) as

TABLE III

TIME AND MONEY SAVINGS OF TRANSIT GROUPS FOR SELECTED GROUP TRANSIT SCHEMES. $\Delta/\text{TRANSIT}$ IS THE AVERAGE TRANSIT DELAY FOR ONE SHIP AND $\text{SAVINGS}/\text{YEAR}$ EXPRESS THE AMOUNT OF US DOLLARS SAVED BY REDUCING THE GROUP TRANSIT DELAYS.

GTS	Schedule	$\Delta/\text{transit}$	Savings/year
current	[10.0 12.0 14.0 16.0 18.0]	2.21 h	0 USD
opt. 4	[10.0 12.7 14.6 16.3]	2.67 h	-9 583k USD
opt. 5	[10.0 12.1 14.0 15.7 17.1]	2.08 h	2 708k USD
opt. 6	[10.0 11.7 13.0 14.3 15.7 17.1]	1.68 h	11 047k USD
opt. 8	[10.0 11.7 13.0 14.0 14.8 15.7 16.9 18.1]	1.19 h	21 250k USD

well as the relative frequency of ships of given speeds (i.e., spacing of speed levels should be tighter for most frequent ship speeds—observable in solutions for the normal speed distribution).

3) *Real-world Impact Assessment*: We compare optimal solutions with the currently deployed GTS on the real-world dataset in terms of time and money saved for the yearly traffic through the Gulf of Aden. We compute delay caused by the GTS only inside the corridor during the transit, not the delay caused by the need to slow down prior the corridor transit. We assume that 20,000 ships sail through the Gulf of Aden in one year and cost of 25,000 USD for one day of shipping [20]. The results are summarized in the Table III.

From the Table III we can observe that for 5 groups, the optimum scheme – compared to the current scheme – saves over 100 shipping days a year, equaling to over 2,5 million US dollars. And even a small change – i.e. adding one speed group level – can reduce the average time for half an hour and save more than 10 million US dollars per year. However, when having a smaller number of escorting ships and the number of speed levels would be reduced from 5 to 4, the additional delay could cause increased costs over 9 million US dollars per year to the shipping industry.

C. On-demand GTS Evaluation

We compare 4 variants of the algorithm designed in Section V.

- 1) *AggregationApproach* – considering constraints of the original problem statement (aggregation at point II and approach zone considered).
- 2) *Approach* – considering only approach constraints, as described in subsection V-C3 (no aggregation point II; ships are spread through the approach zone).
- 3) *Aggregation* – considering only aggregation constraints, as described in subsection V-C4 (aggregation at point II; all ship starting at point A).
- 4) *None* – considering no aggregation and no approach constraints, as described in subsection V-C5 (no aggregation point II; all ship starting at point A).

We look at the structure of the solution, we explore the Pareto frontier, and we evaluate the scalability of the algorithm with respect to main algorithm parameters. Finally, we compare the on-demand group transit to the currently deployed fixed schedule.

We generate synthetic scenarios where ships are spread uniformly in the approach zone which should imitate the

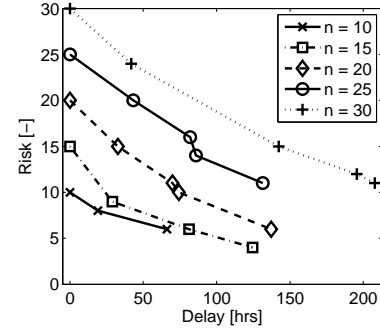


Fig. 7. Pareto frontiers for number of ships $n = \{10, 15, 20, 25, 30\}$. Solutions which are not Pareto efficient, would lie above the curve, as the criterion is minimized.

hardest possible conditions (i.e. the worst case) for successful grouping. Note that in the real-world, the ships would be often significantly clustered and will be grouped farther from the entry point A, thus the solutions would be significantly better (in terms of number of ships grouped). We generate ship speeds from a uniform distribution, again assuming minimum information available and thus emulating the worst case possible. If not stated otherwise, tested values of parameters are set by default to: length of the approach zone $L_a = 600$ nm, number of ships $n = 25$, number of groups $s = 5$, minimum approach speed $v = 8$ kn, maximum speed difference in a group $\Delta V = 2$ kn and minimum group size $\mu = 2$.

1) *Structure of the Solution*: The structure of the solution differs significantly when computed by each algorithm for default parameter setting and for a very high risk weight $\gamma = 1000$.

The solution is displayed in Figure 6. We can observe that for all algorithms except *None*, some ships are left ungrouped (red squares in plots). In case of the *AggregationApproach* algorithm, the ships which are either slow or very close to the entry point A, cannot slow down too much to meet the minimum approach speed constraint. For the *Approach* algorithm, not all ships can be grouped because of the minimum approach speed constraint as well.

Some groups in solutions computed by *AggregationApproach* and *Aggregation* algorithms have slower speed than the speed of the slowest ship in the group. This is caused by the restriction of a specific set of times of arrival U at the aggregation point II.

2) *Pareto Frontier Evaluation*: In the optimization problem defined by Criterion (11), we weigh two functions through a parameter γ . While varying γ , we reach different solutions with different risk and delay caused by the on-demand GTS. Figure 7 captures Pareto frontiers computed by *AggregationApproach* algorithm for default parameter values for $n = \{10, 15, 20, 25, 30\}$ ships. The Pareto frontier curves are almost linear with non-smooth transitions due to the binary property of group membership.

3) *Scalability*: The performance of the algorithm depends on number of parameters: number of ships, number of groups, risk aversion coefficient, maximum speed difference in one group, minimum approach speed and minimum group size. We

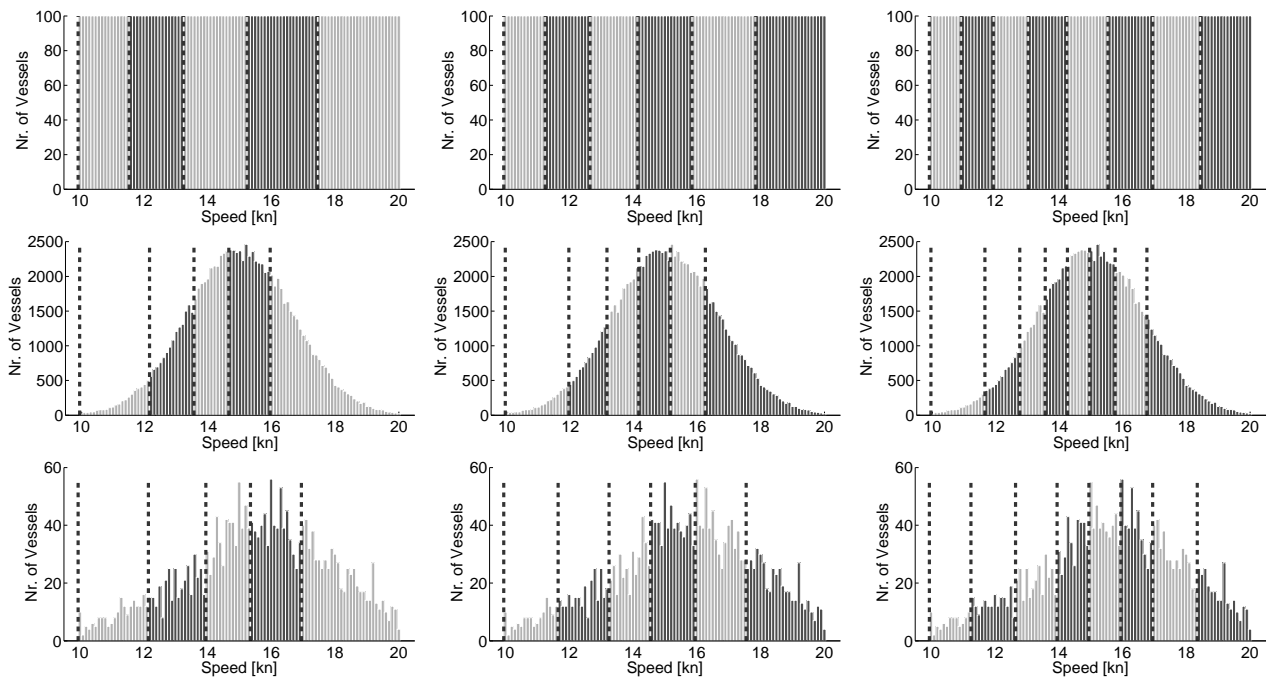


Fig. 5. Optimal groupings for five (left), six (middle) and eight (right) speed levels for three different distributions: uniform (top row), normal (middle row) and real-world (bottom row). The dashed lines denote the speed levels, the different tones of gray scale distinguish between neighboring groups.

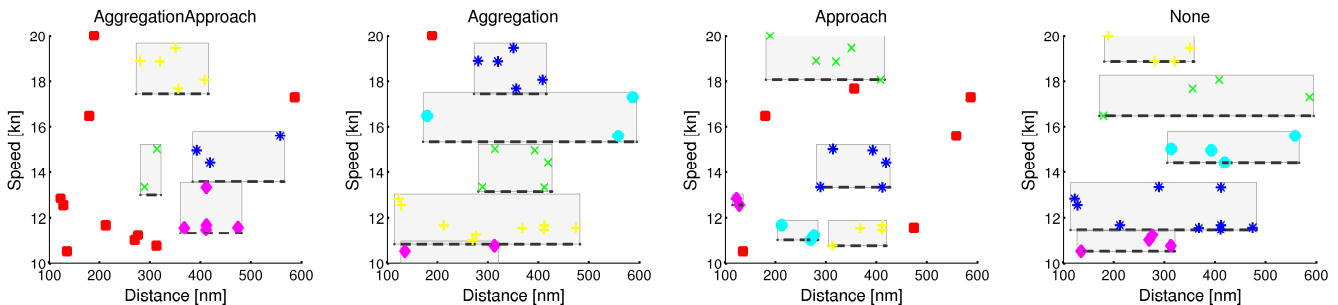


Fig. 6. Structure of Solutions for different constraint sets. Each ship is denoted as a point, the distance of each ship from the entry point $|P_j - A|$ is on the x-axis, the speed of the ship V_j is on the y-axis. The ships grouped into one group are confined within a gray rectangle and dashed lines denote speeds of the groups. Note that for the *Aggregation* and the *None* algorithm, the distance dimension (x-axis) is ignored during the computation. The ships which are not assigned to any group are denoted as red squares, the ships with the same color and marker type belong to the same group.

focus on the first three parameters as they have the greatest impact on the speed of the algorithm. We have generated 100 random instances of the problem for each parameter setting and averaged the computation time needed to find a solution. The dependency of the solution time on the number of ships and the number of groups is depicted in Figure 8. For all 4 algorithms, we are able to find solutions for up to 20 ships and 4 groups. For the *Aggregation* algorithm, when increasing the number of ships or number of groups, some generated problem instances are not solvable within 4GB of RAM.

For the current IRTC transit constraints, the *AggregationApproach* algorithm is able to find a solution for 25 ships and 5 groups in under 10 minutes in average. The largest problems instances solved in hundreds of minutes by *AggregationApproach* were with 30 ships and 6 groups, however, in some cases, the memory was the bottleneck and a solution could not be found.

The scalability of the algorithms with respect to the risk

weight coefficient γ is captured in Figure 9. We varied γ from 0 to 600 for a setting with $n = 20$ ships and $s = 5$ groups, while having all other parameters fixed at default values. For larger risk weights (i.e. $\gamma > 600$), the time needed to find a solution does not vary significantly anymore, as the risk summand in the criterion significantly outweighs the delay summand.

Algorithms without aggregation constraints peak for intermediate values of γ (*Approach* for $\gamma = 200$ and *None* for $\gamma = 80$). After this peak, when increasing γ , the computations time is lower again, converging to a constant value. Algorithms with aggregation constraints (*AggregationApproach* and *Aggregation*) do not have the property described above and the time needed to compute a solution monotonically converges to a constant value.

4) *Comparison with Current Schedules*: To compare the on-demand GTS with the current GTS, we measured the delay caused and the number of ships left ungrouped (by either

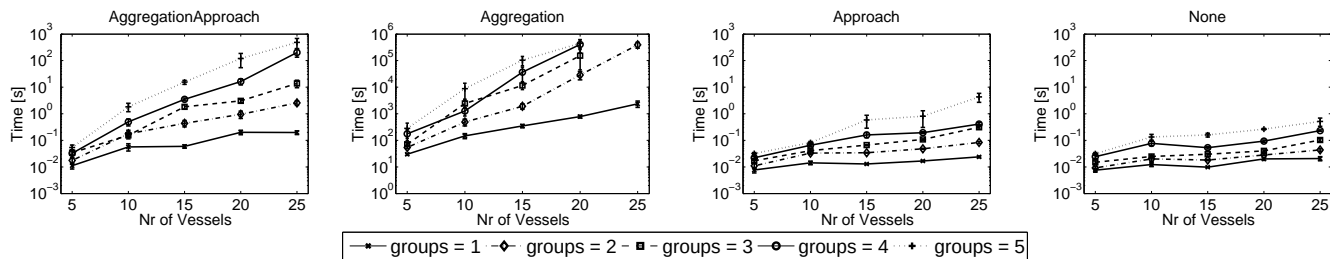


Fig. 8. Scalability of Algorithms while modifying the number of ships and the number of groups. For the *Approach* algorithm, the y-axis is in a different range. Note that the error bars depict standard error.

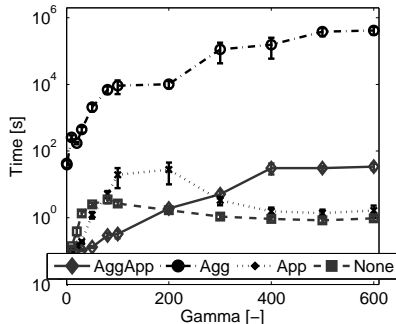


Fig. 9. Scalability of the algorithms with respect to risk weight coefficient γ . Note the significantly lower performance of the *Aggregation* algorithm.

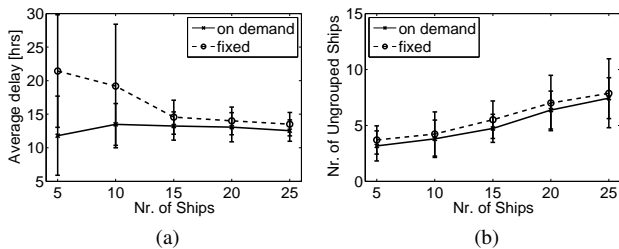


Fig. 10. (a) Comparison of delays caused by the currently deployed GTS and by the on-demand GTS (lower is better); (b) comparison of number of ungrouped ships for fixed and on-demand GTS (lower is better). The error bars depict standard deviation.

not satisfying the minimum approach speed constraint or—in case of the current GTS—being alone in the group) for both the grouping transit schemes. We have set the minimum approach speed to $\nu = 8$ for both schemes. For the on-demand GTS, we have set the risk weight coefficient to $\gamma = 1000$ to force maximum grouping with a possibly increased delay, the number of groups to be created to $s = 5$ and the maximum speed difference in one group to $\Delta V = 2$ to have the parameters equal the currently deployed fixed GTS. We have varied the number of approaching ships from 5 to 25 and run 100 samples for each setting.

The results are depicted in Figure 10. We can observe, that the delay caused by the on-demand GTS is lower for less ships, however, as the number of approaching ships increases, the delay caused by both groupings is comparable, as all groups are filled and given constraints imposed by the aggregation

point II, the delay caused is converging to a stable value⁷. The average time saved by the on-demand grouping with respect to the current GTS is approximately 1 hour, equaling to approximately 8% of the transit time. In Figure 10b, we can compare the average number of ungrouped ships for both schemes. The on-demand GTS has steadily lower number of ungrouped ships (which is preferable). For a high number of ships, the ungrouped ships typical don't meet the minimum approach speed requirement.

VII. CONCLUSIONS

The currently deployed group transit scheme in the ITRC corridor in the Gulf of Aden causes significant delays in shipping and increases shipping costs; it is thus important to deploy such a GTS which minimizes the delay caused. In this work, we first propose a scalable algorithm which transforms the problem of the optimal fixed GTS design to a problem of finding a minimum cost path in a directed acyclic graph; the algorithm is able to compute the optimal GTS for a various number of speed levels in polynomial time. The quality of computed solutions is tested on a dataset containing speed records of ships transiting the Gulf of Aden; the dataset is freely available to the community. Additionally, we quantify possible improvements over the currently deployed GTS when deploying different group transit schedules—we estimate savings to be over 2.5M USD when deploying GTS with the same number of speed levels and over 10M USD when deploying GTS with one additional speed level.

Second, we design a set of mathematical programs able to find the optimal on-demand group transit scheme for a cluster of approaching ships while adhering to constraints posed by the original grouping mechanism. The formulation contains a bi-criterion function balancing the trade-off between the delay caused by the grouping mechanism and the increased risk taken by the ungrouped ships. Compared with the currently deployed GTS, the on-demand GTS is superior both in terms of the delay caused (over 8% lower delay in average) and the number of ships grouped (over 7% more ships grouped in average). The scalability of the mathematical program computing the optimal on-demand GTS is limited, scaling to

⁷Observe rather high standard deviation for 5 and 10 ships which is caused by a low number of groups created. In many cases, there is only a single group; for a single group, the delay can be either high or low, depending on the speed level of the group. For a higher number of ships, a higher number of groups is created and the delay averages out over all speed levels of all the groups.

30 ships and 6 groups. As it can be seen from the evaluation of simplified on-demand grouping versions, relaxation will not always speed up the solution process. Heuristics-based approaches or stochastic optimization techniques may provide solutions for large problems and are the subject of the current research. However, to evaluate the quality of such algorithms, a method able to compute optimal solutions, which is presented in this paper, is crucial.

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