Online Routing Games and the Benefit of Online Data

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ABSTRACT

The goal of this paper is to define and analyze the model of online routing games in order to be able to determine how we can measure and prove the benefits of online real-time data in applications like navigation by autonomous cars. Based on the models of algorithmic game theory and online mechanisms, we define the formal model of online joint resource utilization games and, as their specific version, the model of online routing games. We study simulation runs of online routing games in a Braess network, and define three different notions of the benefit of online data. We outline the possibility of different classes of online routing games and start to investigate the class of simple naïve online routing games that represents the current commercial navigation systems. We prove that in the class of simple naïve online routing games stability is not guaranteed. We prove that in simple naïve online routing games if a single flow enters the network, then the flow on some edge inside the network at some time may be bigger than the one that entered the network. As a consequence we prove that in simple naïve online routing games the worst case benefit of online data may be bigger than one, i.e. it may be a "price". By defining these frames, we open new theoretical research opportunities for important application fields.

Categories and Subject Descriptors

I.2.11 [**Computing Methodologies**]: Distributed Artificial Intelligence – *Multiagent systems.*

General Terms

Performance, Design, Economics, Experimentation, Theory.

Keywords

online routing games, benefit of online data

1. INTRODUCTION

Five ongoing trends became more and more accomplished in the history of computing: ubiquity, interconnection, intelligence, delegation and human-orientation [1]. The current wave of this progress is marked by the widespread availability of online real-time data which opens new possibilities in several application areas like real-time manufacturing intelligence, industrial internet, internet of things, emergency and disaster information services, and intelligent road transport systems. The most challenging applications are those where autonomous agents have access to online real-time data and create plans how to achieve their goals in an environment where they jointly utilize resources that become more costly as more agents use them. In these applications agents are dynamically arriving and departing when they complete their plans. The plans are created by exploiting online data that describe the current status and the current cost of the resources. There is uncertainty about the feasible decision of an agent, because the cost of the resources will change by the time the agent starts to use them: departing agents will release the resources as they complete their plans, agents simultaneously creating their plans will influence each other's costs, and agents arriving later may also influence the costs of the resources used by agents already executing their plans. Because game theory is an appropriate foundation for multi-agent systems [2], we call this type of applications as *online joint resource utilization games* which we will define in this paper. Note that these games are different from resource allocation or minority games [4] which are simultaneous one shot or repeated simultaneous games where there might be some coordination among some of the agents. In contrast, online joint resource utilization games are continuous and non-cooperative games exploiting real-time online data.

A well-known online joint resource utilization game is car navigation using real-time data. In this environment traffic participants generate and use online traffic information to create their own self-optimized plan for their route which contain road sections jointly utilized with other agents. Car navigation using real-time data is a special case of online joint resource utilization games, because the allowed order of the resources in the plan of the agents is restricted by the structure of the road network. From theoretical point of view we call real-time data based car navigation applications as *online routing games*. Note that in our approach each driver makes an individual real-time data based decision when it enters the network, whereas in other approaches [5] drivers learn the best route to select, based on past selections.

Two well-known examples of real-time information based navigation systems are Google Maps and Waze. The planning in these systems is done on central server(s) which may play similar role to the virtual environment in the anticipatory vehicle routing of [3]. There are other traffic management systems that combine central planning and local freedom, like the PLANETS system [12] in which global control strategy is provided from a Traffic Management Centre, but traffic participants have a freedom to make decisions autonomously. In our view, self-interested agents will not conform to a central strategy if it is not individually rational, so the global strategy must emerge from the agents' decision. Therefore we will not have in our model an explicit concept of a central planner or a virtual environment even if the agents use the services of these abstractions.

Although it is widely believed and intuitively we might think that traffic route planning results in shorter travel time if we take into account the real-time traffic information (for example congestions), but no theoretical study is known if real-time data based car navigation produces better traffic or not. There is need for such theoretical studies, because autonomous cars are being designed and the usage of online navigation systems based on the simple naïve strategy discussed in this paper is spreading and we do not even know how to measure their benefit, not to mention how to optimize their behavior. The goal of this paper is to define and analyze the model of online routing games in order to be able to prove the possible benefits of online real-time data.

Neither online joint resource utilization games nor online routing games have been formally defined and studied in detail, so we are advancing the state of the art with the work of this paper. There is related research on routing games without online information in the algorithmic game theory field and there is research on auctions with online information in the online mechanisms field. We are combining these two fields to create the new theoretical model.

Algorithmic game theory [11] studies routing games in which end users simultaneously select a full route to their destination in a network that is susceptible to congestion. It is well known that individually self-optimizing travel routes does not necessarily result in optimal traffic (optimal for the sum of the travel times) and each participant may have longer travel time than with central planning. This is known as the "*price of anarchy*" which was explored by 2012 Gödel prize winners Roughgarden and Tardos. In their paper [9] they investigated the old conundrum in transportation science known as the "Braess's paradox" [10]. The algorithmic game theory investigations revealed important properties of routing games, however the algorithmic game theory approach includes assumptions which do not handle the dynamic online information environment.

Dynamic agent systems have been studied in the framework of online mechanisms, like in [7]. Online mechanisms [8] extend mechanism design to dynamic environments where agents continuously enter and leave the environment. Agents in online mechanisms make decisions without knowing the future. Agents have a type which is described by their time of arrival, time of departure and their valuation of the resources to be allocated. The utility of the agents is the difference between their valuation and the cost of the allocated resource. While online mechanism design is good for certain types of dynamic environments, its assumptions do not handle the decentralized nature of real-time data based car navigation systems. Here decentralization refers to both decision making by all the agents and cost value determination by the resources at the actual time of resource utilization. In online mechanisms the cost of the allocated resource is determined at negotiation time by a centralized agent, while in real-time data based car navigation the final cost is determined by the resources when the resource is actually used, and the final cost may be different from the one at decision time.

In order to be able to forecast the behavior of the dynamic agent environments of online joint resource utilization games and to be able to measure and prove the benefits of online real-time data, our contributions are the following:

- Based on the models of algorithmic game theory and online mechanisms, we define the first model of online joint resource utilization games and, as their specific version, the model of online routing games.
- We study the simulation of an online routing game in a Braess network and point out that there are worst, best and average benefits of online real-time data, and we define these notions. We outline the possibility of different classes of online routing games with different strategies.
- We prove three properties of the class of the simple naïve strategy online routing games and the benefit of online real-time data in these games.

With these advances we open new research opportunities for important application fields, define the main characteristics that can form the basis to guide future research and help to compare the results of future research contributions.

2. Related Work

In this section we highlight the main theoretical findings of routing games and online mechanism design, because the first model of online joint resource utilization games presented in this paper is based on them and the research in the new field of online joint resource utilization games has to answer similar questions.

2.1 Routing Games

Algorithmic game theory studies networks with source routing (section 18 in [11]), in which end users simultaneously choose a full route to their destination and the traffic is routed in a congestion sensitive manner. Two models are used: nonatomic selfish routing and atomic selfish routing. Nonatomic routing is meant to model the case when there are very many actors, each controlling a very small fraction of the overall traffic. Atomic routing is meant to model the case when each actor controls a considerable amount of traffic. Both models are studied in detail and showed similar properties. The main difference is that different techniques are required for their analysis, because the nonatomic model basically has continuous functions having unique extreme values, while the atomic model has discrete functions approximating the extreme values at several points.

The algorithmic game theory model of the routing problem is the (G, r, c) triple, where

- G is the road network given by a directed graph G=(V, E) with vertex set V and edge set E;
- r is the total traffic flow given by a vector of r_i traffic flows with r_i denoting the amount of flow on the P_i trip which is from the s_i source vertex of G to the t_i target vertex of G; and
- c is the throughput characteristic of the road network given by a cost function with c_e: $R^+ \rightarrow R^+$ for each e edge of G mapping the total traffic on edge e to the travel time on that edge.

In this model the G graph may contain parallel edges; the c_e cost functions are nonnegative, continuous and nondecreasing; the r_i traffic flow on the P_i trip is deterministically routed somehow on the paths leading from s_i to t_i; the cost of a path is the sum of the costs of the edges in the path at a given flow; and the cost of an r_i traffic flow on the P_i trip is the sum of the cost of all the paths in P_i. In the case of a nonatomic routing problem the r_i traffic flow on the P_i trip may be divided arbitrarily among several paths leading from s_i to t_i, while in the case of an atomic routing problem the r_i traffic flow on the P_i trip can be sent on one single path leading from s_i to t_i. We assume that in the routing problem each actor is interested in an r_i traffic flow on a P_i trip, therefore we will use the term "actor", "agent" and "r_i traffic flow" interchangeably.

A flow distribution is optimal if it minimizes the cost of the total traffic flow over all possible flow distributions. A flow distribution is an equilibrium flow distribution if none of the actors can change its traffic flow distribution among its possible paths to decrease its cost. The equilibrium flow distribution is a rational choice for every autonomic actor, because deviating from the equilibrium would increase the cost for the actor.

It is proven (section 18 in [11]) that every nonatomic routing problem has at least one equilibrium flow distribution and all equilibrium flow distributions have the same total cost. The price of anarchy is the ratio between the cost of an equilibrium flow distribution and the optimal flow distribution. If the cost functions are of the form $ax+b$, then the price of anarchy in any nonatomic routing problem is not more than $4\div3$. If the cost functions can be nonlinear, then one can create cost functions to exceed any given bound on the price of anarchy of nonatomic routing problems.

In atomic routing problems the existence of equilibrium flow distribution is not always guaranteed. Atomic routing problems have equilibrium flow distribution if every r_i traffic flow has the same value or if the cost functions are of the form $ax+b$. If there are more than one equilibrium flow distributions, then their total costs may be different. If the cost functions of an atomic routing problem are of the form $ax+b$, then the price of anarchy is at most $(3+\sqrt{5})\div2$. If the cost functions of an atomic routing problem are of the form $ax+b$ and in addition every r_i traffic flow has the same value, then the price of anarchy is at most $5\div2$.

It is known that if the routing problem has an equilibrium and the actors try to minimize their own cost (best-response), then the traffic flow distribution converges to an equilibrium.

The algorithmic game theory investigations of the routing game revealed important properties, however the algorithmic game theory model contains the following assumptions:

a) the throughput characteristic of the network does not change with time and the drivers can compute this characteristic or learn it by repeatedly passing the road network;

b) the drivers simultaneously decide their optimal route; and

c) the outcome travel time for a given driver depends on the choice of all the drivers and the characteristic of the network, but not on the schedule of the trip of the drivers.

These assumptions do not completely describe car traffic where the drivers use car navigation with online data, in which case

a) the throughput characteristic of the network changes with time and the drivers cannot compute or learn this characteristic by repeatedly passing the road network, because there may be an accident on a road and the road becomes susceptible to congestion, and then later when the accident is cleared the road is less susceptible to congestion;

b) the drivers do not decide their route at the same time simultaneously, because drivers continuously enter the road network and decide their optimal route when they enter the road network and the decision is based on the current live information about the status of the road network; and

c) the outcome travel time for a given driver depends not only on the current characteristic of the network and the route choice of all the drivers currently entering the road network, but also on the trip schedule of other drivers that entered the network previously, are currently entering the network or will enter the network later.

The issue of traffic dynamism is studied in the field of dynamic traffic assignment [6], but there they investigate the time-varying properties of traffic flow, whereas here we assume that the traffic flow is constant and only the cost functions may change. In our investigations the critical issue is the sequential decision making of the agents. This partly handled by online mechanisms.

2.2 Online Mechanisms

Online mechanism design problem is a multi-agent sequential decision making problem. When agents participate in the mechanism, they report to a central planner for a given period their request for certain resources at given valuations (which may be different from their private values). The central planner decides which resources at which cost are allocated to which agent in each time step. All agents are trying to maximize their utility.

The model of the online mechanism problem is the (t, Θ, k, c, u) five tuple, where

$t=\{1, 2, \dots\}$ is a possibly infinite sequence of time periods;

Θ is the set of agent types where each agent type is characterized by the $\theta_i=(a_i,d_i,v_i(t))$ triple where $a_i\in t$ is the arrival time of the agent, $d_i\in t$ is the departure time of the agent, and $v_i(t)$ is the valuation function of the agent in time period $t\in[a_i,d_i]$, the agent has no value for $t\notin[a_i,d_i]$;

$k=(k^1, k^2, \dots)$ is a sequence of decision vectors with $k^t=(k^t_1, k^t_2, \dots)$ decision vector made in time period t and k^t_i the decision made for agent a_i;

c is the cost function of the decisions and $c(k^t_i)$ is the cost for agent a_i in time period t;

u is the utility function of the agents where $u_i(t)=c(k^t_i) - v_i(t)$ is the utility for agent a_i in time period t, and all agent aim to maximize their utilities;

In this model the Θ set of agent types may be model free when no probabilistic information is known about the agents, or may be model based if probabilistic information is known. The agents may report values different from their private agent type, but only for the time period when they are present, at the beginning of the reported time period and without knowing the reports of the other agents (closed direct revelation). Usually the goal is to design online mechanisms where the truthful revelation is the dominant strategy. The effectiveness of online mechanisms is measured similarly as that of online algorithms: the performance of the online mechanism is compared with that of an offline mechanism that has the complete information about all future agent types.

The dynamic nature of online mechanisms is a good starting point to model online joint resource utilization games, however the differences are considerable: in contrast with online mechanisms, there is no central planner (agents make their own plan), there is no arrival and departure time (agents want to start their plan when they arrive), and the actual cost is determined not at the decision time, but at utilization time.

3. The Model of Online Joint Resource Utilization Games

In order to have a generic model, we are now defining the model of the online joint resource utilization game as an extension of the algorithmic game theory model of the routing problem and the online mechanisms. The model resembles the algorithmic game theory routing game model in the concepts of flow, cost and resource, and it resembles the model of online mechanisms in the sequences of time periods and decisions. T time unit is introduced in order to be able to compute the rate of resource utilization.

The model of the online joint resource utilization game is the (t, T, G, c, r, k) sextuple, where

t={1, 2, …} is a possibly infinite sequence of time periods;

T is a natural number, T time periods give one time unit;

G is a set of resources where each $e \in G$ resource is characterized by a c_e cost function and t_e resource utilization time;

c is the cost function of the resources with $c_e: R^+ \rightarrow R^+$ for each $e \in G$ mapping the resource utilization flow (the total number of agents starting to use the e resource from time period t-T to time period t) to the current cost of the resource;

r is the total resource utilization flow given by a vector of r_i resource utilization flows with r_i denoting the resource utilization flow (number of agents in T time periods) for the P_i plan which is a set of vectors containing elements from G;

$k=(k^1, k^2, \dots)$ is a sequence of decision vectors with $k^t=(k^t_1, k^t_2, \dots)$ decision vector made in time period t and k^t_i the decision made by the agents of the r_i resource utilization flow in time period t.

In this model the c_e cost functions are nonnegative, continuous and nondecreasing; the c_e cost functions have a fixed minimum value plus a flow dependent part, where the flow dependent part is not known to any of the agents of the model and the agents can learn the actual cost only when an agent finishes using the resource and reports its cost; the r_i resource utilization flow is given by $T \div n_i$ where n_i is a natural number constant, meaning that one agent enters the game in each cycle of n_i time periods (0 agent entering the game in time periods 1, 2, … n_i-1 and 1 agent entering the game with the goal of P_i plan in the n_i time period); the k^t_i decision is to instantiate the P_i plan to one of its vectors; and the actual cost of a plan instantiation (e_1, e_2, e_3, …) for a flow starting at time period t is $c_{e1}(t)+c_{e2}(t+t_{e1})+c_{e3}(t+t_{e1}+t_{e2})+\dots$, i.e. the actual cost of a resource is determined at the time when the usage of that resource starts.

3.1 The Model of Online Routing Games

We are now defining the model of online routing games as online joint resource utilization games with a restriction on the allowed plans represented by a graph and with somewhat different cost functions. The typical application of online routing games is real-time data based car navigation where the graph represents a road network, the agents represent the cars, resource utilization means passing a road section and the cost of resource utilization represents the travel time on a road section.

The model of the online routing game is the (t, T, G, c, r, k) sextuple, where

t, T and k are the same as in online joint resource utilization games;

G is a directed graph G=(V, E) with vertex set V and edge set E where each $e \in E$ is characterized by a c_e cost function which is equal to its utilization time;

c is the cost function of G with $c_e: R^+ \rightarrow R^+$ for each e edge of G mapping the flow to the travel time on that edge, but not less than the remaining cost of any other agents currently utilizing that edge increased with the time gap of the flow of the agent[1];

r is the total flow given by a vector of r_i flows with r_i denoting the flow aiming for a P_i trip from a s_i source vertex of G to a t_i target vertex of G;

In this model the G graph may contain parallel edges. The c_e cost functions are nonnegative, continuous and nondecreasing, their variable part are not known to any of the agents of the model and the agents can learn the actual cost only when an agent exits an edge and reports it; the r_i flow is given by $T \div n_i$ where n_i is a natural number constant. The k^t_i decision is how the P_i trip is routed on a single path of the paths leading from s_i to t_i and the actual cost of a path (e_1, e_2, e_3, …) for a flow starting at time period t is $c_{e1}(t)+c_{e2}(t+c_{e1}(t))+c_{e3}(t+c_{e1}(t)+c_{e2}(t+c_{e1}(t)))+\dots$, i.e. the actual cost of an edge is determined at the time when the flow enters the edge.

The agents can learn the actual cost of the edges only when an agent finishes using the resource and reports its cost. Because agents do not report cost values in each time step, the agents interested in the cost values must do reasoning and decrease the last reported value by taking into account the time elapsed since the last reporting event (it is like pheromone evaporation in [16]).

The online routing game model can accommodate changes of the c cost function over the t sequence of time periods, because the agents can get information about the actual cost only from the cost reported by the agents exiting an edge.

3.1.1 Routing strategy

The critical point in the online routing game is how to determine the best decision vector k. The algorithmic game theory approach assumes that the agents have full information about the cost functions and the theory tells what the best strategy is in the case of simultaneous decisions, but does not tell how the agents can achieve this. In online mechanisms a central planner decides which resources at which cost are allocated to which agent. In online routing games there is no central planner. The agents in online routing games will have to apply algorithms similar to online algorithms [13]. At this time we are not investigating how the agents of online routing games determine their strategy, instead we are investigating how current navigation systems perform in online routing games.

In practice, typical navigation software in cars use simple shortest path search in the road network, possibly modifying the distances with the online information about the actual traffic delay. We call this decision strategy as *simple naïve strategy*. We are investigating this strategy in this paper because of its practical importance. Note that the simple naïve strategy is by definition deterministic, thus it is a pure strategy.

Although online routing game has some resemblance to a sequence of atomic routing problems of the algorithmic game theory approach, online routing game is much more complex than the atomic routing problem and we do not know any theoretical results regarding the existence of equilibrium flow or something like the price of anarchy in online routing games.

[1] In this model cars cannot overtake the cars already on the road and there is a time gap, i.e. minimum "following distance".

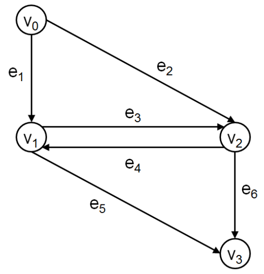

Figure 1. The SN_{Braess} network.

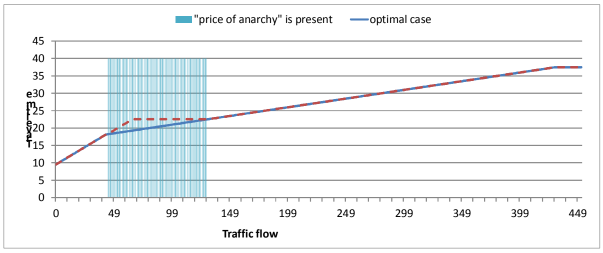

Figure 3. Cost in the SN_{Braess} network depending on the flow using the algorithmic game theory approach.

4. Simulation of an Online Routing Game

Several authors investigated with simulation tools how the traffic would behave if the majority of vehicles used traffic information in their route planning and concluded that online data has to be used carefully in traffic scenarios [15]. These investigations did not have theoretical conclusions. In order to have a better understanding of online routing games and formally prove what others suspected from empirical investigations, we are demonstrating simulation runs in a small instance of online routing games which we call Simple Naïve Braess (SN_{Braess}) online routing game, because it investigates the real-time data based car navigation problem on a network corresponding to the Braess paradox [10] and the decision mechanism uses the simple naïve strategy. We selected the SN_{Braess} online routing game, because the simple naïve strategy has practical importance and because the Braess paradox is a distinguished study area of transportation science and algorithmic game theory. The analysis of simulation runs in this paper is not a full statistical analysis, because the goal is to understand the behavior of the agents.

The SN_{Braess}=(t, T, G, r, c, k) instance of online routing games used in the simulations has the following concrete values

T=600, modeling one minute.

G is the road network (shown in Figure 1.) is given as a four node directed graph with V=(v_0, v_1, v_2, v_3) and E=(e_1, e_2, e_3, e_4, e_5, e_6). The edges are $e_1=v_0\rightarrow v_1$, $e_2=v_0\rightarrow v_2$, $e_3=v_1\rightarrow v_2$, $e_4=v_2\rightarrow v_1$, $e_5=v_1\rightarrow v_3$, and $e_6=v_2\rightarrow v_3$. Note that the e_4 edge is included to allow bidirectional travel between v_1 and v_2 and to have an uncongested route from the source to the destination.

r=(r_1) is the total traffic flow with only one flow on the P_1 trip from the v_0 source vertex of G to the v_3 target vertex of G;

c is the cost function of the road network with $c_{e1}=1+x\div10$, $c_{e2}=15$, $c_{e3}=7.5$, $c_{e4}=7.5$, $c_{e5}=15$, $c_{e6}=1+x\div10$, where x is the total number of agents entering an edge from time period t-T to time period t. As discussed in the previous section, the variable part of the cost function is not known to the agents of the model.

k=(k^1, k^2, …) is a sequence of decision vectors with $k^t=(k^t_1)$ decision vector made in period t. The decision is a simple naïve decision mechanism which is based on the currently reported costs and selects the path on the P_1 trip that currently has the minimum reported cost. The P_1 trip has the {p_1, p_2, p_3, p_4} set of paths, where p_1=(e_1,e_5), p_2=(e_1,e_3,e_6), p_3=(e_2,e_6), p_4=(e_2,e_4,e_5) and the costs of the paths used for decision making are the sum of the cost of the edges at the time of decision making, e.g. $c_{p1}(t)=c_{e1}(t)+c_{e5}(t)$.

From algorithmic game theory point of view the Braess road network (without online data) is used to demonstrate the "paradox" that the equilibrium flow has a price of anarchy with at most $4\div3$ ratio. The "paradox" is that if nonatomic selfish flow allocation is used, then at some flow values the e_3 edge increases the total cost ("price of anarchy"), and without the e_3 edge (seemingly smaller throughput) the cost is smaller (faster travel). The cost values (in this case travel times) of the trips in the SN_{Braess} network from the algorithmic game theory point of view are shown with red dashed line in the diagram of Figure 3. as a function of the incoming traffic flow. The figure shows the optimal case of central planning (blue line) as well. When there is a difference, then "price of anarchy" is present.

We have investigated the cost values in the SN_{Braess} online routing game by simulation runs. A constant traffic flow was entered into the network from time zero and the history of the travel time of the cars arriving at their destination at node v_3 during a simulation run of 411000 time periods were recorded in a diagram like in Figure 2. These travel time diagrams revealed different patterns depending on the amount of traffic flowing into the network. In some cases they had higher frequency variation with some sudden peaks, while in other cases the variations seemed to be smoother with some disturbance periods. Figure 2. shows the travel time diagram at incoming traffic flow of 120, which represents a heavy load in the "price of anarchy" range in the algorithmic game theory approach. This diagram, like the others, has a starting phase where the travel time is increasing as more and more cars are on the road. Later, as the shortest path gets congested, the drivers deviate and keep deviating to other routes, so the travel time does not seem to converge to a constant value. The travel time varies between 16 and 46.5 with an average of 30.73.

Note that in this SN_{Braess} road network a) if the routes are planned with optimal central control, then the travel time is 22.0, b) if the routes are planned in accordance with the algorithmic game theory approach, then the travel time is 22.5 (with price of

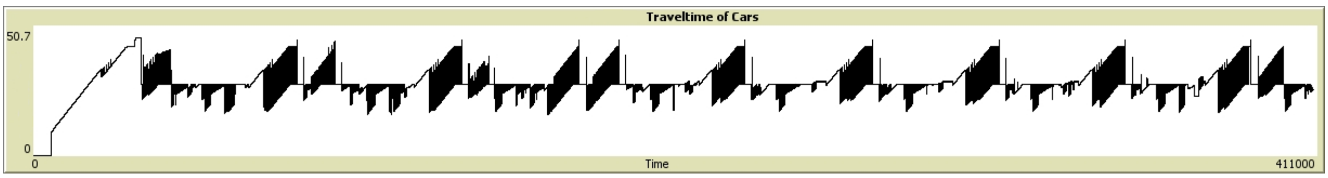

Figure 2. The diagram of the travel time of the cars in the SN_{Braess} road network at incoming traffic flow 120.

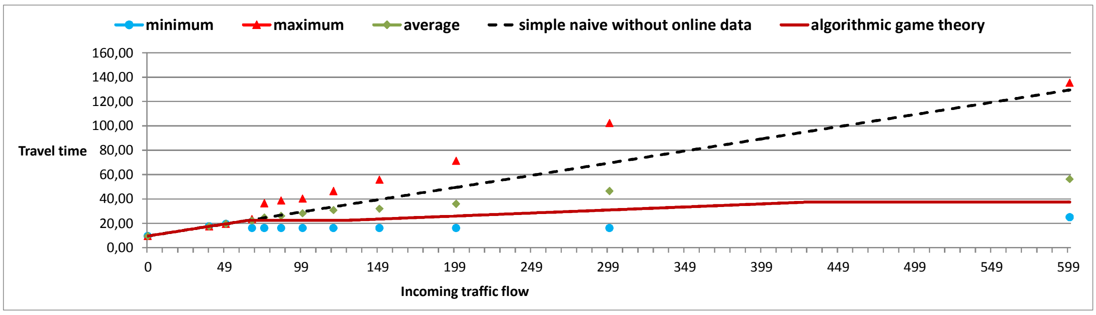

Figure 4. Maximum, minimum and average travel times in the SN_{Braess} road network at selected incoming traffic flows.

anarchy), c) if the routes are planned with simple naïve strategy without real-time congestion delay information (i.e. all cars select the p_2 path), then the travel time is 33.5, and d) if all cars select the p_4 path which is not susceptible to congestion and is the longest path without congestion delay, then the travel time is 37.5.

We cannot include too many travel time diagrams in this paper, instead we sum up the measured maximum, minimum and average travel time values at some selected incoming flows in graphical form on Figure 4. This figure includes the travel time for the algorithmic game theory approach (continuous red line) and the travel time for the simple naïve strategy that does not use online data (dashed black line).

We can observe that at low incoming traffic flows the cars do not deviate from the shortest path, so the computed and the SN_{Braess} travel times are the same. At higher flows there is fluctuation and the cars sometimes experience fast traffic, but often they experience considerable congestion delay and travel longer than the travel time of the simple naïve strategy without real-time congestion delay information.

5. The Benefit of Online Real-time Data

We would like to be able to tell if the agents are better off by making decisions based on online real-time data or not. In order to be able to compare the costs of the agents using online data with the costs of the agents not using online data, we have to know what we are going to compare with what.

Oracle based evaluation

If we take the approach of online algorithms, then we would compare the results of the online routing game with the results of an oracle that has all the information needed. One might think that in our case the oracle with all information would be the central planning[2], because the central planner has all the information and can tell each agent which route to take. The central planner produces the travel times as shown by the blue line in Figure 3.

The central planning oracle might be good to measure the global effectiveness of the agents in the SN_{Braess} model, however it evaluates not only the benefits of making decisions based on online data, but in addition it evaluates the different decision making strategies as well. In the SN_{Braess} model there is no coordination among the agents and the agents make online decisions using the simple naïve strategy, while in the central planning and the algorithmic game theory approach the agents are coordinated and exploit their knowledge about the cost functions. Therefore if we want to evaluate only the benefits of online real-time data, then we want to compare the results with an "oracle" using the same decision making strategy.

Oracle with the same decision making strategy

The decision making strategy of the SN_{Braess} model is to select the path with the smallest cost (shortest travel time) using the current online real-time data and the agents do not have a cost model of the network. The corresponding decision strategy without online real-time data is the one which uses the shortest path without the variable part of the cost functions. The dashed black line in Figure 4. shows this oracle. As we can see in Figure 4. the agents of the SN_{Braess} model perform much better in some cases (minimum values), in average (average values) they perform better at higher incoming traffic flow range and sometimes in the worst case (maximum values) they perform worse, mainly in the medium flow range.

Best, worst and average

In the algorithmic game theory model there is equilibrium and the price of anarchy concept is the ratio between the equilibrium and the optimum. In the simulation runs of the SB_{Braess} model there are maximum, minimum and average cost values at every incoming traffic flow. Later in this paper, in Theorem 1 we will prove that there are simple naïve strategy online routing games which do not have equilibrium at some flow values. If there is no equilibrium, then we must have different measures of the benefit of online real-time data for the best, worst and average cases (which are guaranteed to exist if there are finite sequence of time periods).

Depending on the type of application, we are interested in the different types of benefits. The most important is the worst case, because it can be used to provide a guarantee in critical applications. The best case can be used in applications, where we have to make sure that a certain value is achieved at least once. The average case is seldom useful in itself, usually we have to consider other statistical distribution parameters as well.

Measure of the benefit of online real-time data

We conclude the above discussion with defining the different benefits of online real-time data. If these benefits are greater than 1, then they are in fact a "price" like the price of anarchy.

Definition 1. The *worst case benefit* of online real time data at a given flow is the ratio between the cost of the maximum cost of the flow and the cost of the same flow with an oracle using the

[2] It is assumed that the central planner is able to produce the optimal flow distribution as defined on page 463 in [11].

same decision making strategy and only the fixed part of the cost functions.

Definition 2. Similarly, the *best case benefit* of online real time data at a given flow is the ratio between the cost of the minimum cost of the flow (after the initial running up) and the cost of the same flow with an oracle using the same decision making strategy and only the fixed part of the cost functions.

Definition 3. The *average case benefit* of online real time data at a given flow is the ratio between the cost of the average cost of the flow and the cost of the same flow with an oracle using the same decision making strategy and only the fixed part of the cost functions.

6. Classes of Online Routing Games

In the above sections we have seen that the decision strategy is important in online routing games. The aim is to select a decision strategy that results in costs close to the optimum. Although the above discussed simple naïve decision strategy is often applied in real world, it is not the best, because it does not alternate the agents of a flow among two or more paths, whereas the optimal central planning and the algorithmic game theory approach use several paths for the same flow.

Further research is needed to study different online routing game decision strategies derived from other related games. In this paper we are only mentioning a possibility. In addition to the already discussed shortest path planning, we can start the development of decision strategies from resource allocation or minority games [4]. These games are simultaneous games containing some coordination as well, but in online routing games coordination is excluded. The coordination aspect could be replaced by some probabilistic value and each agent would randomly select its path in the network based on a probabilistic distribution. The optimal probabilistic distribution values could be determined using a kind of evolutionary algorithm like the one applied to the El Farol Bar problem in [14].

Online routing games using the same type of decision strategies belong to the same class of online routing games. Each class need to be evaluated how much benefit they make out of online real-time data, in order to be able to determine the type of application where they are suitable. The evaluation should include formal proofs. In this paper we are formally analyzing the class of simple naïve strategy online routing games.

7. Three Properties of the Class of Simple Naïve Strategy Online Routing Games

We are now proving three properties of the simple naïve strategy online routing games.

THEOREM 1. *There are simple naïve strategy online routing games which do not have equilibrium at certain flow values.*

PROOF. We show some games that satisfy this claim. Let $SN_{7.1}$ be a (t, T, G, r, c, k) simple naïve strategy online routing game. Let $r=(r_1)$ be the total traffic flow with only one flow on the P_1 trip of G. Let $P_1=\{p_1,p_2\}$ with p_1 and p_2 two different paths of P_1. Let c_{nc1} be the cost of p_1 when there is no congestion on p_1 and c_{c1} be the cost of p_1 when r_1 flows on p_1. Similarly let c_{nc2} be the cost of p_2 when there is no congestion on p_2 and c_{c2} be the cost of p_2 when r_1 flows on p_2. There might be cost functions such that $c_{c1}>c_{c2}>c_{nc1}>c_{nc2}$ at r_1 flow. In this case there is no equilibrium, because in the beginning p_2 is selected by all agents, but as soon as the cost of p_2 goes above c_{nc1}, all agents select p_1, so p_2 becomes less congested and, as a result, the cost of p_2 will drop below the cost of p_1, so all agents will select p_2 again, and the cycle starts again. □

THEOREM 2. *There are SN=(t, T, G, r, c, k) simple naïve strategy online routing games where the total traffic flow has only one incoming flow, i.e. $r=(r_1)$, however the flow on some of the edges of G sometimes may be more than r_1.*

PROOF. We show some games where this is possible. Let $SN_{7.2}$=(t, T, G, r, c, k) be a simple naïve strategy online routing game. Let $r=(r_1)$ be the total traffic flow with only one flow on the P_1 trip of G. Let $P_1=\{p_1,p_2\}$ with p_1 and p_2 two different paths of P_1. Let $p_1=(e_1,e_3)$, $p_2=(e_2,e_3)$ be the edges of the paths. Let e_2 be an edge not susceptible to congestion, c_2 be the cost of e_2, and $c_2>2\times T$. Let e_3 edge be susceptible to congestion. Let the e_1 edge be susceptible to congestion and let the cost of e_1 be such that c_{nc1} is the cost of e_1 when there is no congestion on e_1 and c_{c1} is the cost of e_1 when r_1 flows on e_1, and $1.5\times c_2>c_{c1}>c_2>c_{nc1}$. This is possible at some r_1 flow on the edge for example if the cost function is linear. When r_1 starts to flow into G, then it goes on the $p_1=(e_1,e_3)$ path, because $c_2>c_{nc1}$. In the beginning the cost of e_1 is c_{nc1}, but it is increasing and at some t_{xc1} time (where $t_{xc1}<T$) the cost of e_1 reaches c_2. The c_2 travel time will be reported to other agents when these agents exit e_1, i.e. at $t_{xc1}+c_2$ time. From this time agents will select the $p_2=(e_2,e_3)$ path. This flow from e_2 will reach the e_3 edge at $t_{xc1}+c_2+c_2$ time. We are going to show that at this time the flow from e_1 to e_3 already reached the r_1 value and it is still r_1. At time T the agents are still selecting the e_1 edge, because $T<0.5\times c_2$ and $0<t_{xc1}$, so $T< t_{xc1}+c_2$, the path change time. Therefore the flow from e_1 to e_3 does reach the r_1 value at $T+c_{c1}$ time. Because $T<0.5\times c_2$, $c_{c1}<1.5\times c_2$ and $0<t_{xc1}$, we get $T+c_{c1}<t_{xc1}+c_2+c_2$, so the flow from e_1 to e_3 already reached the r_1 value when agents start to flow from e_2 to e_3. The last agent selects the e_1 edge at $t_{xc1}+c_2$ time and because $0<t_{xc1}$ and $T<0.5\times c_2$, this agent starts after T at full r_1 flow therefore the cost for this agent is c_{c1}. Therefore the flow from e_1 to e_3 does not stop until $t_{xc1}+c_2+c_{c1}$. Because $c_2<c_{c1}$, we get $t_{xc1}+c_2+c_2<t_{xc1}+c_2+c_{c1}$, so the flow from e_1 to e_3 still has the r_1 value when agents start to flow from e_2 to e_3. So the e_3 edge will receive r_1 flow from the e_1 edge and some additional flow from the e_2 edge at $t_{xc1}+c_2+c_2$ time, and the cost of the agents coming from the e_1 edge is c_{c1}. (Note that it is enough if G contains a sub-graph as described in the proof of Theorem 2 and the r_1 flow continuously flows into this sub-graph.) □

THEOREM 3. *There are SN=(t, T, G, r, c, k) simple naïve strategy online routing games where the worst case benefit of online real-time data is greater than one, i.e. in these games the worst case benefit is a "price".*

PROOF. Let us take the Let $SN_{7.2}$=(t, T, G, r, c, k) simple naïve strategy online routing game example from Theorem 2 and let the cost of e_3 be such that c_{nc3} is the cost of e_3 when there is no congestion on e_3, c_{c3} is the cost of e_3 when r_1 flows on e_3. We know from Theorem 2 that there is some time when agents with c_{c1} cost from the e_1 edge will flow into the e_3 edge with r_1 flow, and at the same time another flow will flow into the e_3 edge, so the cost of these agents on the e_3 edge will be some c_{c3+}, which is greater than c_{c3}, because the cost functions are non-decreasing. So the total cost of the agents on the p_1 path will be $c(p_1)=c_{c1}+c_{c3+}$. The oracle with simple naïve strategy without online data would send all the time all traffic on p_1 with $c(p_1)=c_{c1}+c_{c3}$ cost, so the worst case benefit of online real-time data is $(c_{c1}+c_{c3+})\div(c_{c1}+c_{c3})$. Because $c_{c3+}>c_{c3}$, the worst case benefit of online real-time data is greater than one at this flow. □

8. CONCLUSION

Information and communication technologies allow that modern car navigation systems utilize live online data from traffic networks to optimize the route of autonomic vehicles. Several authors investigated with simulation tools how the traffic would behave if the majority of autonomic vehicles based their route planning on such navigation systems and concluded that online data has to be used carefully in traffic scenarios. In order to be able to measure and prove properties of traffic routing based on online data, we have defined the formal model of online joint resource utilization games and, as their specific version, the model of online routing games. To our knowledge, we are the first to define these models. These models are extensions of the models of routing games of the algorithmic game theory approach and the online mechanisms. Based on the formal model, we analyzed the simulation of a simple routing scenario in a Braess network and pointed out the different aspects of the benefit of online data, and defined three notions of the benefit of online data. We foresee different classes of online routing games, among them the class of simple naïve online routing games currently applied in commercial products. We proved that in the class of simple naïve online routing games stability is not guaranteed, so it makes sense to talk about worst, average and best benefit of online data. We proved that in simple naïve online routing games it may happen that a single flow enters the network and on some edge inside the network the flow is bigger than the one that entered the network. As a consequence we proved that the worst case benefit of online data may be bigger than one, i.e. it may be a "price". These results are in line with previous simulation results, but now we have given formal proofs.

With these advances we opened new research opportunities for important application fields and determined the main notions and characteristics that can become the basis to guide future research. We challenge future research to develop online routing game decision strategies that have worst case benefits of online data below one, or prove that it is not possible to develop such strategies. If such strategies are possible, then we expect that the application of these new strategies will be individually rational choice and therefore the decision strategies can be implemented in the navigation devices themselves instead of the centralized planning approaches like those of Google Maps and Waze, because some users are reluctant to provide private data for the centralized approach.

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